Cosmological perturbations in extended massive gravity

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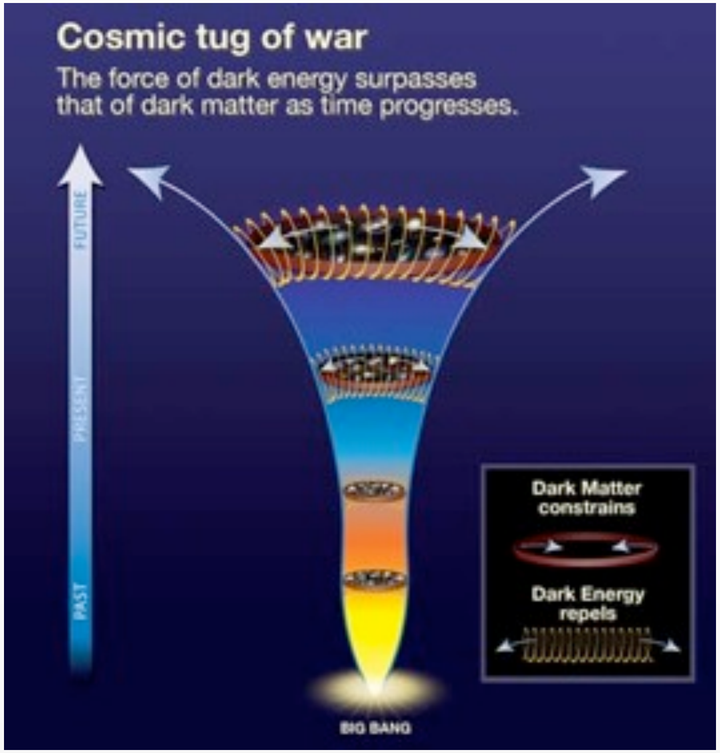
Mini-workshop "Massive gravity and its cosmological implications"

Outline

- Introduction
- Quasi-dilaton
- Varying mass gravity theory

Introduction



Cosmic acceleration

Introduction

- Can we give graviton a mass?
  - Fierz and Pauli 1939

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$\mathcal{L}_{FP} = f^4 \left(h_{\mu\nu} h_{\mu\nu} - h^2\right)$$

van Dam-Veltman-Zakharov discontinuity

$$T^\mu_\nu h^\nu_\mu = T^\mu_\nu (\hat{h}^\nu_\mu + m_g^2 \delta^\nu_\mu \phi) = T^\mu_\nu \hat{h}^\nu_\mu + \frac{1}{M_{\rm Pl}} T\phi^c$$

  - Vainshtein 1972 non-linear interactions
  - Boulware-Deser (BD) ghost 1972

Lack of Hamiltonian constrain and momentum constrain

6 degrees of freedom

Helicity ±2, ±1, 0 $\longrightarrow$ 5 dof?

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Lack of Hamiltonian constrain and momentum constrain

$\Downarrow$

6 degrees of freedom

Helicity ±2, ±1, 0 $\longrightarrow$ 5 dof?

6th dof is the BD ghost!

Introduction

- C. de Rham, G. Gabadadze and A. Tolly 2011

$$I_g = M_{Pl}^2 \int d^4x \sqrt{-g} \left[\frac{R}{2} + m_g^2 (\mathcal{L}_2 + \alpha_3 \mathcal{L}_3 + \alpha_4 \mathcal{L}_4) \right]$$

$$\mathcal{K}^\mu_\nu = \delta^\mu_\nu - \left(\sqrt{g^{-1} f} \right)^\mu_\nu \qquad f_{\mu\nu} \equiv \eta_{ab} \partial_\mu \phi^a \partial_\nu \phi^b$$

$$\mathcal{L}_2 = \frac{1}{2} ([\mathcal{K}]^2 - [\mathcal{K}^2]) , \qquad [\mathcal{K}] \equiv Tr\mathcal{K}$$

$$\mathcal{L}_3 = \frac{1}{6} ([\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2[\mathcal{K}^3]) ,$$

$$\mathcal{L}_4 = \frac{1}{24} ([\mathcal{K}]^4 - 6[\mathcal{K}]^2[\mathcal{K}^2] + 3[\mathcal{K}^2]^2 + 8[\mathcal{K}][\mathcal{K}^3] - 6[\mathcal{K}^4]) ,$$

4 Stukelberg scalars respect Poincare symmetry

$$\phi^a \to \phi^a + c^a , \qquad \phi^a \to \Lambda^a_b \phi^b$$

$$\langle f \rangle \neq 0$$

Source of MASS !

Eliminate a helicity-0 mode, the so called BD ghost in the decoupling limit.

$$\mathcal{K}_{\mu\nu} = \partial_\mu \partial_\nu \pi \longrightarrow \mathcal{L}_2, \mathcal{L}_3, \mathcal{L}_4 \text{ all become total derivative}$$

It is also BD ghost free away from decoupling limit. (Hassan & Rosen 2011)

Introduction

- FRW solutions

  arXiv:1109.3845...et.al...

- Ghost instability
  - Linear perturbation: vanishing kinetic term
  - Non-linear perturbations:

    A new ghost instability found

- Towards healthy massive cosmologies
  - Relax FRW symmetry
  - **Extended massive gravity theory**

Quasi-dilaton

- The simplest extension with dilaton-like global symmetry

$$\sigma \to \sigma - \alpha M_{\rm Pl}\,, \qquad \phi^a \to e^\alpha \phi^a$$

$$S = \frac{M_{\rm Pl}^2}{2}\int d^4x\,\sqrt{-g}\,\Big[R - 2\Lambda + 2\,m_g^2\left(\mathcal{L}_2(\bar{\mathcal{K}}) + \alpha_3\,\mathcal{L}_3(\bar{\mathcal{K}}) + \alpha_4\,\mathcal{L}_4(\bar{\mathcal{K}})\right) - \frac{\omega}{M_{\rm Pl}^2}\,\partial_\mu\sigma\,\partial^\nu\sigma + \mathcal{L}_{\rm matter}\Big]\,,$$

The building block tensor

$$\mathcal{K}^\mu_\nu \to \bar{\mathcal{K}}^\mu_\nu \equiv \delta^\mu_\nu - e^{\sigma/M_{\rm Pl}}\left(\sqrt{g^{-1}f}\right)^\mu_\nu\,.$$

Quasi-dilaton

- We choose the Minkowski fiducial metric and FRW physical metric

$$f_{\mu\nu} = -n^2(t)\delta^0_\mu\delta^0_\nu + \delta_{ij}\delta^i_\mu\delta^j_\nu$$

$$g_{\mu\nu}dx^\mu\, dx^\nu = -dt^2 + a^2(t)\,\delta_{ij}dx^i\, dx^j$$

The eom of Stuckelberg scalar yields

$$(1-X)\,X\,\left[3+3\,\alpha_3+\alpha_4-(3\,\alpha_3+2\,\alpha_4)\,X+\alpha_4\,X^{\,2}\right] = \frac{\text{constant}}{a^4}$$

There are 4 attractors

$$X \equiv \frac{e^{\sigma/M_p}}{a}\,.$$

$X=0$, $X=1$ and $X=X_\pm$ with

$$X_\pm = \frac{3\,\alpha_3+2\,\alpha_4 \pm \sqrt{9\,\alpha_3^2-12\,\alpha_4}}{2\,\alpha_4}$$

Quasi-dilaton

- The Friedmann equation

$$\left(3 - \frac{\omega}{2}\right) H^2 = \Lambda + \Lambda_\pm \,,$$

$$\Lambda_\pm = -\frac{m_g^2}{2\,\alpha_4^3}\left[9\,(3\,\alpha_3^4 - 6\,\alpha_3^2\,\alpha_4 + 2\,\alpha_4^2) \pm \alpha_3\,(9\,\alpha_3^2 - 12\,\alpha_4)^{3/2}\right]$$

sensible cosmology requires

$$\omega < 6$$

The eom for dilaton $\left.\frac{\delta S}{\delta \sigma}\right|_{f=t} = 0.$

$$-\frac{X}{M_p \omega a^3 N}\left.\frac{\delta S}{\delta \sigma}\right|_{f=t} = \frac{1}{N}\frac{d}{dt}\left(\frac{\dot{X}}{N}\right) + 3\,H\,X\left(H + \frac{\dot{X}}{N\,X}\right) + X\left[\frac{\dot{H}}{N} - \left(\frac{\dot{X}}{N\,X}\right)^2\right]$$
$$+\frac{m_g^2 X^2}{\omega}\left[3r(1-2X) - 6X + 9 + 3(X-1)(r(3X-1) + X - 3)\alpha_3 - (X-1)^2(r(4X-1)-3)\alpha_4 - 4\,r\,X^3\,\xi\right] = 0.$$

Quasi-dilaton

- Perturbations

$$g_{\mu\nu} = g^{(0)}_{\mu\nu} + \delta g_{\mu\nu} \ ,$$

$$\delta g_{00} = -2\,N^2\,\Phi \, ,$$
$$\delta g_{0i} = N\,a\,\left(B_i^T + \partial_i B\right) \, ,$$
$$\delta g_{ij} = a^2\left[h_{ij}^{TT} + \frac{1}{2}(\partial_i E_j^T + \partial_j E_i^T) + 2\,\delta_{ij}\,\Psi + \left(\partial_i\partial_j - \frac{1}{3}\delta_{ij}\,\partial_l\partial^l\right)E\right] \, ,$$

Where $\partial^i h_{ij}^{TT} = h_i^{TT\,i} = 0, \quad \partial^i B_i^T = 0, \quad \partial^i E_i^T = 0 \, .$

Then we introduce the perturbations for scalar

$$\sigma = \sigma^{(0)} + M_p\,\delta\sigma \ .$$

In our calculations, we choose unitary gauge, and write action in Fourier plane waves

$$\vec{\nabla}^2 \to -k^2, \qquad d^3x \to d^3k.$$

Quasi-dilaton

- The tensor mode

$$S_{\rm T} = \frac{M_{\rm Pl}^2}{8} \int d^3k\, a^3\, dt \left[|\dot{h}_{ij}^{TT}|^2 - \left(\frac{k^2}{a^2} + M_{GW}^2 \right) |h_{ij}^{TT}|^2 \right]$$

where

$$M_{GW}^2 \equiv \frac{m_g^2 (r-1) X^3}{X-1} + H^2 \omega \left(\frac{r}{r-1} + \frac{2}{X-1} \right) \qquad r \equiv \frac{a}{N}.$$

generally,

$$M_{GW} \sim \mathcal{O}(H)$$

so even if the tensor modes are tachyonic, the time scale of their instability is of the order of the age of the universe.

Quasi-dilaton

- The vector mode

$$S_{\rm V}=\frac{M_{\rm Pl}^2}{16}\int d^3k\,a^3\,dt\,k^2\left[\frac{|\dot{E}_i^T|^2}{\left(1+\frac{k^2(r^2-1)}{2a^2H^2\omega}\right)}-M_{GW}^2|E_i^T|^2\right]$$

  - In the case $(r^2-1)/\omega \geq 0$,
  - if $(r^2-1)/\omega < 0$, critical momentum scale, $k_c = aH\sqrt{\frac{2\omega}{1-r^2}}$,

    $\Lambda_{UV}^2\,(1-r^2)/(H^2\,\omega) < 2.$
  - Canonical normalize the vector perturbations

$$\mathcal{E}_i^T \equiv \frac{M_{\rm p}}{2}\,\mathcal{T}_V\,E_i^T\,,$$

$$S=\frac{1}{2}\int d^3k\,a^3\,N\,dt\left(\frac{1}{N^2}|\dot{\mathcal{E}}_i^T|^2-\omega_V^2|\mathcal{E}_i^T|^2\right),$$

Quasi-dilaton

The dispersion realation

$$\omega_V^2 = (1+q^2)\, M_{GW}^2 - \frac{H^2\, q^2 (1+4\, q^2)}{(1+q^2)^2}\,, \qquad q^2 \equiv \frac{k^2}{a^2}\, \frac{r^2-1}{2\, H^2\, \omega}\,.$$

- The 2nd term is fine, if the 1st term is positive
- On the other case if $M_{GW}^2 < 0$ and $q^2 > 0$.

$$\Lambda_{UV}^2 \lesssim \frac{2\, H^2\, \omega}{r^2-1}\,,$$

Quasi-dilaton

- The scalar mode

$$\delta g_{00} = -2\,N^2\,\Phi\,, \qquad \delta g_{0i} = N\,a\,\partial_i B\,,$$
$$\delta g_{ij} = a^2\left[2\,\delta_{ij}\,\Psi + \left(\partial_i\partial_j - \frac{1}{3}\delta_{ij}\,\partial_l\partial^l\right)E\right], \qquad \sigma = \sigma^{(0)} + M_p\,\delta\sigma\,.$$

After integrating out the non-dynamical mode and the would-be BD ghost mode, we have

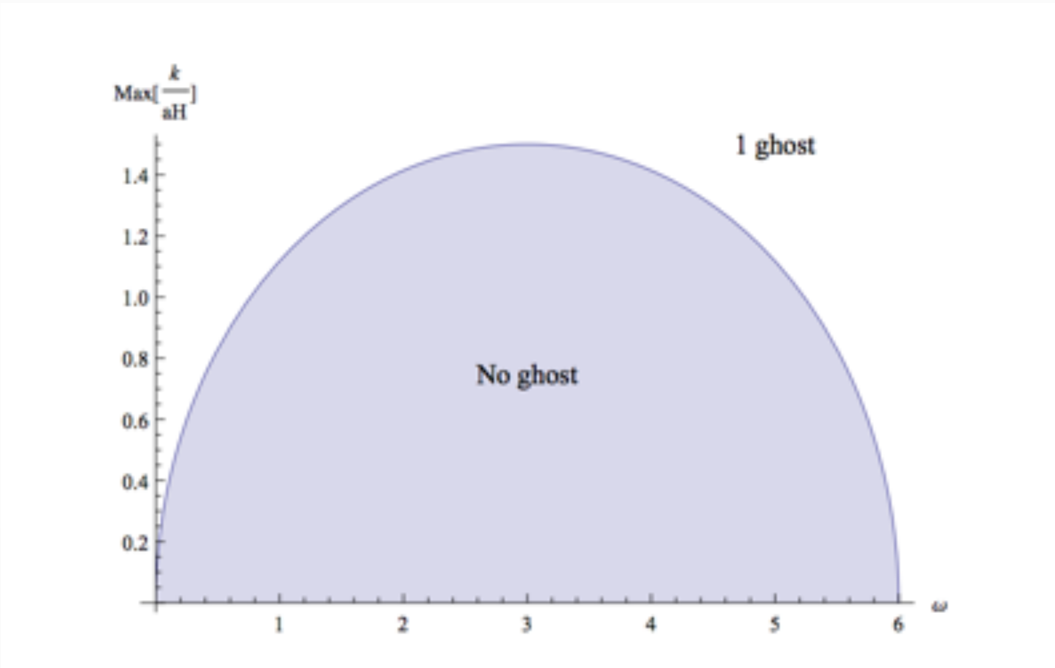
$$S = \int \frac{d^3k}{2}\,a^3\,N\,dt\left[\frac{\dot{Y}^\dagger}{N}\mathcal{K}\frac{\dot{Y}}{N} + \frac{\dot{Y}^\dagger}{N}\mathcal{M}\,Y + Y^\dagger\,\mathcal{M}^T\,\frac{\dot{Y}}{N} - Y^T\,\Omega^2\,Y\right]$$

where $\quad Y \equiv (\tilde{\delta\sigma},\ E), \qquad \tilde{\delta\sigma} = \frac{1}{\sqrt{2}\,k^2}\,(\Psi - \delta\sigma)$

Quasi-dilaton

it is sufficient to study the determinant

$$\det \mathcal{K} = \frac{3\, M_p^4\, k^6\, \omega^2\, a^4\, H^4}{\left[\omega\, a^2\, H^2 - \frac{4\,k^2}{6-\omega}\right]\, (r-1)^2}$$



Along with $0 < \omega < 6\,,$

No ghost requires

$$\frac{k}{a\,H} < \frac{\sqrt{\omega(6-\omega)}}{2}\,.$$

At least 1 ghost subhorizon

Quasi-dilaton

- There is one extra term, but it doesn't change the kinetic determinant

$$\int d^4x \sqrt{-f}\, e^{4\sigma/M_p},$$

- Higher derivative terms, the kinetic metric receive order 1 modification, but it is still negative

$$(\partial\sigma)^2(\partial^2\sigma)^n + \cdots$$

Varying mass gravity

- Promote the graviton mass parameter to a function of a scalar

$$S = \int d^4x \sqrt{-g} \left\{ \frac{M_p^2}{2} \left[R[g] - 2\Lambda + 2m_g^2(\sigma) \left[\mathcal{L}_2 + \alpha_3(\sigma) \mathcal{L}_3 + \alpha_4(\sigma) \mathcal{L}_4 \right] \right] - \frac{1}{2} \partial_\mu \sigma \, \partial^\nu \sigma - V(\sigma) \right\}$$

$$\mathcal{K}^\mu_\nu = \delta^\mu_\nu - \left(\sqrt{g^{-1} \bar{g}} \right)^\mu{}_\nu .$$

$$\bar{g}_{\mu\nu} dx^\mu dx^\nu = -n(t)^2 dt^2 + \alpha(t)^2 \delta_{ij} dx^i dx^j .$$

The eom for stuckelburg scalars

$$\frac{m_g^2 (X-1)}{X^3} \left[3 - 3 (X-1) \alpha_3 + (X-1)^2 \alpha_4 \right] = \text{constant},$$

$$X \equiv 1/a \text{ and } r \equiv an.$$

The time dependence of m_g and $\alpha_{3,4}$ allows the nontrivial Cosmological solutions.

Varying mass gravity

- The background eoms

$$3\,H^2 = \Lambda + \frac{1}{M_{\rm Pl}^2}\left(\rho_\sigma + \rho_m\right)\,, \quad \dot{H} = -\frac{1}{2\,M_{\rm Pl}^2}\left[\left(\rho_\sigma + p_\sigma\right) + \left(\rho_m + p_m\right)\right]$$

$$\dot\rho_m + 3\,H\left(\rho_m + p_m\right) = -Q\,, \quad \dot\rho_\sigma + 3\,H\left(\rho_\sigma + p_\sigma\right) = Q\,.$$

$$\begin{aligned}
\rho_m &\equiv M_{\rm Pl}^2 m_g^2\,(X-1)\,\left[6 + 4\,\alpha_3 + \alpha_4 - X\,(3 + 5\,\alpha_3 + 2\,\alpha_4) + X^2\,(\alpha_3 + \alpha_4)\right]\,,\\
p_m &\equiv M_{\rm Pl}^2 m_g^2\,\big[6 + 4\,\alpha_3 + \alpha_4 - (2+r)\,X\,(3 + 3\,\alpha_3 + \alpha_4)\\
&\qquad\qquad + (1 + 2\,r)\,X^2\,(1 + 2\,\alpha_3 + \alpha_4) - r\,X^3\,(\alpha_3 + \alpha_4)\big]\,,\\
Q &\equiv M_{\rm Pl}^2\,m_g^2\,\dot\sigma\,(X-1)^2\bigg\{\alpha_3'\,(4 - X - 3\,r\,X) + \alpha_4'(X-1)\,(r\,X-1)\\
&\qquad + \frac{2\,m_g'}{m_g}\left[3 - (X-1)\alpha_3 + \frac{r\,X-1}{X-1}\left[3 - 3\,(X-1)\,\alpha_3 + (X-1)^2\alpha_4\right]\right]\bigg\}\,,\\
\rho_\sigma &\equiv \frac{\dot\sigma^2}{2} + V\,, \quad p_\sigma \equiv \frac{\dot\sigma^2}{2} - V\,, \qquad\qquad (68)
\end{aligned}$$

where prime denotes differentiation with respect to σ.

Varying mass gravity

- Tensor perturbation

$$\delta g_{ij} = a^2 h_{ij}^{TT} \,,$$

$$S = \frac{M_p^2}{8} \int d^3k \, a^3 \, N \, dt \left(\frac{1}{N^2} |\dot{h}_{ij}^{TT}|^2 - \left(\frac{k^2}{a^2} + M_{GW}^2 \right) |h_{ij}^{TT}|^2 \right)$$

where

$$M_{GW}^2 = \frac{(r-1)\, X^2}{(X-1)^2} \left[m_g^2 \, (X-1) - \frac{\rho_m}{M_p^2} \right] - \left(\frac{1}{r-1} + \frac{2\, X}{X-1} \right) \frac{\rho_m + p_m}{M_p^2} \,.$$

The stability of long wavelength tensor modes is ensured by $M_{GW}^2 > 0$.

Varying mass gravity

- Vector perturbations

$$\delta g_{0i} = N\,a\,B_i^T\,, \qquad \delta g_{ij} = \frac{a^2}{2}\left(\partial_i E_j^T + \partial_j E_i^T\right),$$

$$S_{\rm V} = \frac{M_{\rm Pl}^2}{16}\int d^3k\,a^3\,dt\,k^2\left[\frac{|\dot{E}_i^T|^2}{\left(1-\frac{k^2\,(r^2-1)\,M_{\rm Pl}^2}{2a^2\,(\rho_m+p_m)}\right)} - M_{GW}^2|E_i^T|^2\right]$$

Ghost free for all momentum below cut-off

$$\frac{\Lambda_{UV}^2\,(1-r^2)}{H^2R} < 2, \quad R \equiv -\frac{\rho_m+p_m}{M_{\rm Pl}^2H^2}$$

$$\left[1+\frac{1}{8NH}\frac{d}{dt}\ln\left(\frac{RM_{GW}^2}{r^2-1}\right)\right]\frac{\Lambda_{UV}^2(1-r^2)}{H^2R} < \frac{3}{2}+\frac{1}{4NH}\frac{d\,\ln\left(M_{GW}^2\right)}{dt}$$

Varying mass theory

- Scalar perturbations

$$\delta g_{00} = -2\,N^2\,\Phi\,, \qquad \delta g_{0i} = N\,a\,\partial_i B\,, \qquad \delta g_{ij} = a^2\left[2\,\delta_{ij}\,\Psi + \left(\partial_i\partial_j - \frac{1}{3}\delta_{ij}\,\partial_l\partial^l\right)E\right]$$

while the scalar field is expanded as,

$$\sigma = \sigma^{(0)} + M_p\,\delta\sigma\ .$$

The quadratic action

$$S = \int \frac{d^3k}{2}\,a^3\,N\,dt\left[\frac{\dot{Y}^\dagger}{N}\mathcal{K}\,\frac{\dot{Y}}{N} + \frac{\dot{Y}^\dagger}{N}\mathcal{M}\,Y + Y^\dagger\,\mathcal{M}^T\,\frac{\dot{Y}}{N} - Y^\dagger\,\Omega^2\,Y\right]$$

where $Y \equiv (\tilde{\delta\sigma},\ E)$, $\qquad \tilde{\delta\sigma} = \frac{1}{\sqrt{2}\,k^2}\left(\Psi - \frac{M_p\,H\,N}{\dot{\sigma}}\delta\sigma\right)$

Varying mass

- Define the new variable to diagonalize the matrix

$$Z_1 \equiv \frac{k^3\, M_p}{3\, a\, H} \left[E + 6\sqrt{2} \left(1 - \frac{3\, a^2\, (\rho_m + p_m)}{2\, k^2 M_p^2 (r-1)} \right) \tilde{\delta\sigma} \right] ,$$

$$Z_2 \equiv \frac{k^2\, M_p}{\sqrt{6}} \left(E + 6\sqrt{2}\, \tilde{\delta\sigma} \right) ,$$

The quadratic action can be written as

$$S \ni \int \frac{d^3k}{2}\, a^3\, N\, dt \left(\kappa_1 \frac{\dot{Z}_1^\dagger}{N} \frac{\dot{Z}_1}{N} + \kappa_2 \frac{\dot{Z}_2^\dagger}{N} \frac{\dot{Z}_2}{N} \right) ,$$

$$\kappa_1 = \left[\frac{k^2}{a^2\, H^2 \left(\frac{\rho_\sigma + p_\sigma}{4\, M_p^2\, H^2} - \frac{3}{2} \right)} - \frac{\rho_m + p_m}{M_p^2\, H^2} \right]^{-1} , \qquad \kappa_2 = 1 \, .$$

Varying mass

The momentum should satisfy

$$\left(\frac{\rho_\sigma + p_\sigma}{4\,M_p^2\,H^2} - \frac{3}{2}\right)^{-1} \frac{k^2}{a^2} > \frac{\rho_m + p_m}{M_p^2}$$

de Sitter like expansion, i.e. $|\dot{H}| \ll H^2$,

$$R + \frac{4}{R-6}\frac{k^2}{H^2 a^2} > 0\,, \quad \longrightarrow \quad R > 6\,.$$

Conclusion

- Ghost in quasi-dilaton theory is a robust feature.
- varying mass gravity can be Ghost free